

MIGRATION TRAPS IN DISKS AROUND SUPERMASSIVE BLACK HOLES

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ABSTRACT

Accretion disks around supermassive black holes (SMBHs) in active galactic nuclei contain stars, stellar mass black holes, and other stellar remnants, which perturb the disk gas gravitationally. The resulting density perturbations exert torques on the embedded masses causing them to migrate through the disk in a manner analogous to planets in protoplanetary disks. We determine the strength and direction of these torques using an empirical analytic description dependent on local disk gradients, applied to two different analytic, steady-state disk models of SMBH accretion disks. We find that there are radii in such disks where the gas torque changes sign, trapping migrating objects. Our analysis shows that major migration traps generally occur where the disk surface density gradient changes sign from positive to negative, around $20\text{--}300R_g$, where $R_g = 2GM/c^2$ is the Schwarzschild radius. At these traps, massive objects in the AGN disk can accumulate, collide, scatter, and accrete. Intermediate mass black hole formation is likely in these disk locations, which may lead to preferential gap and cavity creation at these radii. Our model thus has significant implications for SMBH growth as well as gravitational wave source populations.

Subject headings: black hole physics — accretion disks — galaxies:nuclei

1. INTRODUCTION

At present, the observational evidence for intermediate mass black holes (IMBHs; $M \sim 10^2\text{--}10^6 M_\odot$) is much less compelling than that for supermassive black holes (SMBHs; $M > 10^6 M_\odot$) or stellar mass black holes ($M \lesssim 40 M_\odot$). Several IMBH candidates have been identified, including off-nuclear X-ray sources such as HLX-1 (likely $\sim 10^3\text{--}10^5 M_\odot$) (Davis et al. 2011; Servillat et al. 2011; Straub et al. 2014) and optical emission line sources in dwarf galaxies (Reines et al. 2013; Moran et al. 2014; Baldassare et al. 2015). IMBHs are a missing link between stellar-mass black holes and SMBHs, and indeed are good candidates for the seeds of SMBHs (Haiman & Loeb 2001). IMBH candidates are hard to confirm, although they are predicted to be wandering throughout massive galaxy halos (Holley-Bockelmann et al. 2010; Bellovary et al. 2010) or lurking in dwarf galaxies (Wassenhove et al. 2010).

An additional potential habitat for IMBHs is the accretion disks around SMBHs in active galactic nuclei (AGN). Massive objects (stellar remnants and stars) will exist in these disks (Syer et al. 1991; Artymowicz et al. 1993; Levin 2007; Nayakshin & Sunyaev 2007; McKernan et al. 2012), where they can collide, accrete and grow. If a mechanism exists to efficiently collect compact objects into an orbit where they can collide, this mass buildup could result in the efficient formation of IMBHs in AGN disks.

Migration toward trapping orbits may be such a mechanism. Objects orbiting within differentially rotating disks exchange angular momentum with the gas around them as they orbit, which results in a torque, typically causing the objects to migrate. Under the azimuthally isothermal assumption, masses within disks were shown to migrate only inwards (Goldreich & Tremaine 1979; Ward 1997; Tanaka et al. 2002). However, Paardekooper & Mellema (2006) found that in the more realistic case of an adiabatic midplane, migration can proceed outwards under some circumstances. Paardekooper et al. (2010) used an extensive set of numerical simulations to empirically define the conditions determining the sign and strength of migration. Locations where the torque changes sign from positive to negative have outwardly migrating objects meeting inwardly migrating objects in an equilibrium, zero-torque orbit, forming a migration trap. Such traps have been predicted to exist in protoplanetary disks (Lyra et al. 2010), where they can lead to rapid growth of giant planet cores (Horn et al. 2012). McKernan et al. (2012) pointed out that, by analogy, IMBH might be able to form efficiently and grow at super-Eddington rates in SMBH accretion disks, if they contained migration traps. Eventually, the resulting object may be able to clear a gap in the disk, which would produce a range of observational signatures (McKernan et al. 2014).

Here we show that simple, analytic, steady-state models of AGN disks do indeed predict migration traps, at radii that are independent of the SMBH mass and the mass ratio between the migrator and the central SMBH. We further briefly discuss the importance and observational implications of migration traps in AGN disks.

2. METHODS

In this section we describe the torque model of Paardekooper et al. (2010), and discuss its application to two different steady-state AGN accretion disk models.

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2.1. Torque Model

The torque model is based on simulations performed to study the behavior of objects in protoplanetary disks, but the physical processes modeled are no different in optically-thick AGN accretion disks. We assume that the mass of the migrating object (i.e. a stellar mass black hole) remains constant, and neglect accretion or feedback effects on the gas. The torque model includes a linear estimate of the Lindblad (wave) torque plus a simple but nonlinear contribution from adiabatic corotation torques. It is valid for the unsaturated case, where a temperature gradient is maintained by turbulent and viscous diffusion, as opposed to the gradient being erased as angular momentum is transferred between the migrating object and nearby gas. Saturation can be neglected so long as the diffusion timescale is short compared to the libration timescale on which the torque acts (Kley et al. 2009).

We model the torques using the analytical fits of Paardekoooper et al. (2010) to a broad range of simulations that included non-isothermal effects and a nonlinear model of adiabatic corotation torques. For the locally isothermal case, the normalized torque is

$$\Gamma_{\text{iso}}/\Gamma_0 = -0.85 - \alpha - 0.9\beta, \quad (1)$$

while for the purely adiabatic case the normalized torque is

$$\gamma\Gamma_{\text{ad}}/\Gamma_0 = -0.85 - \alpha - 1.7\beta + 7.9\xi/\gamma. \quad (2)$$

The adiabatic index $\gamma = 5/3$, and the variables α , β , and ξ are the negative gradients of the local density, temperature, and entropy, with values

$$\alpha = -\frac{\partial \ln \Sigma}{\partial \ln r}; \beta = -\frac{\partial \ln T}{\partial \ln r}; \xi = \beta - (\gamma - 1)\alpha. \quad (3)$$

The torques are normalized by

$$\Gamma_0 = (q/h)^2 \Sigma r^4 \Omega^2, \quad (4)$$

where q is the mass ratio of the migrator to the SMBH, h is the aspect ratio of the disk, and Ω is the rotational velocity. Interpolating between the isothermal and adiabatic torque regimes, we obtain

$$\Gamma = \frac{\Gamma_{\text{ad}}\Theta^2 + \Gamma_{\text{iso}}}{(\Theta + 1)^2} \quad (5)$$

where Θ is the ratio of the radiative and dynamical timescales $t_{\text{rad}}/t_{\text{dyn}}$. Lyra et al. (2010) show that Θ depends on the local disk properties as

$$\Theta = \frac{c_v \Sigma \Omega \tau_{\text{eff}}}{12\pi \sigma T^3} \quad (6)$$

where c_v is the thermodynamic constant with constant volume, τ_{eff} is the effective optical depth, and σ is the Stefan-Boltzmann constant. The value of τ_{eff} is taken at the midplane (Hubeny 1990; Kley & Crida 2008) as

$$\tau_{\text{eff}} = \frac{3\tau}{8} + \frac{\sqrt{3}}{4} + \frac{1}{4\tau} \quad (7)$$

where τ is the true optical depth, calculated by $\tau = \kappa\Sigma/2$, where κ is the opacity.

In summary, each of the torque components depends on the properties of the temperature, density, and entropy gradients. For particular values of these gradients,

the torques may cancel, resulting in a region with zero torque, i.e. a migration trap. Our goal is to investigate whether stable traps exist, i.e. whether there are regions where the gradient of the torque is negative.

AGN disks are sufficiently ionized for magnetorotational instability to drive turbulence. The resulting density perturbations produce stochastic torques that can drive diffusive, random walk, migration (Nelson 2005). Johnson et al. (2006) quantify when diffusive migration dominates over advective (type I) migration. Simulations of fully-ionized regions of stratified protoplanetary disks suggest that for interesting ranges of migrator mass and radius, type I migration prevails (Yang et al. 2009). Such stochastic perturbations were shown by Horn et al. (2012) to be necessary for multiple objects to reach equilibrium orbits and collide. We defer numerical simulations of the AGN case to future work.

2.2. Disk Models

We examine the torques expected in disks described by two steady-state, analytic SMBH accretion disk models derived by Sirko & Goodman (2003, hereafter SG) and Thompson et al. (2005, hereafter TQM). These models are derived from different basic assumptions, but both contain many characteristics we expect in realistic AGN disks. Neither model includes direct modeling of magnetic fields, nor effects due to general relativity.

SG assume a classical thin Keplerian α -disk (Shakura & Sunyaev 1973) in a steady state with a constant, high, accretion rate (Eddington ratio of 0.5). In order to remain stable and prevent fragmentation (i.e. maintain $Q \gtrsim 1$), SG assume that stars form in the outer disk. Energetic feedback from the newly formed stars increases the velocity dispersion and sound speed of the gas, maintaining Q close to unity, supporting the disk against global gravitational instability and inhibiting further star formation. This approach is supported by the existence of nuclear star clusters in the vicinity of SMBHs, which may have formed in this way (Nayakshin 2006; Levin 2007; Chang et al. 2007). The disk opacity model of SG is based on Iglesias & Rogers (1996) for high temperatures ($T \gtrsim 10^4$ K) and Alexander & Ferguson (1994) for lower temperatures.

The model of TQM, on the other hand, extrapolates a star-forming galaxy disk inward to the SMBH. Angular momentum transport is assumed to take place due to global gravitational instabilities, such as bars and spiral inflows, rather than unresolved turbulent viscosity. TQM use a more up-to-date opacity model based on Semenov et al. (2003). TQM address gravitational fragmentation by considering two regimes: one where the external accretion rate is high enough that the gas fraction of the disk remains constant, allowing rapid inflow to continue; and another where the star formation timescale is shorter than the gas advection time, and thus accretion to the inner regions is more limited, as the gas is consumed in star formation.

Figure 1 shows profiles from both models of the disk temperature T , surface density Σ , aspect ratio h/r , and optical depth τ . Although the profiles are qualitatively comparable, there are major differences between the models. For example, the surface density and optical depth in SG are 2-3 orders of magnitude above those of TQM in the inner disk. The differences in opacity

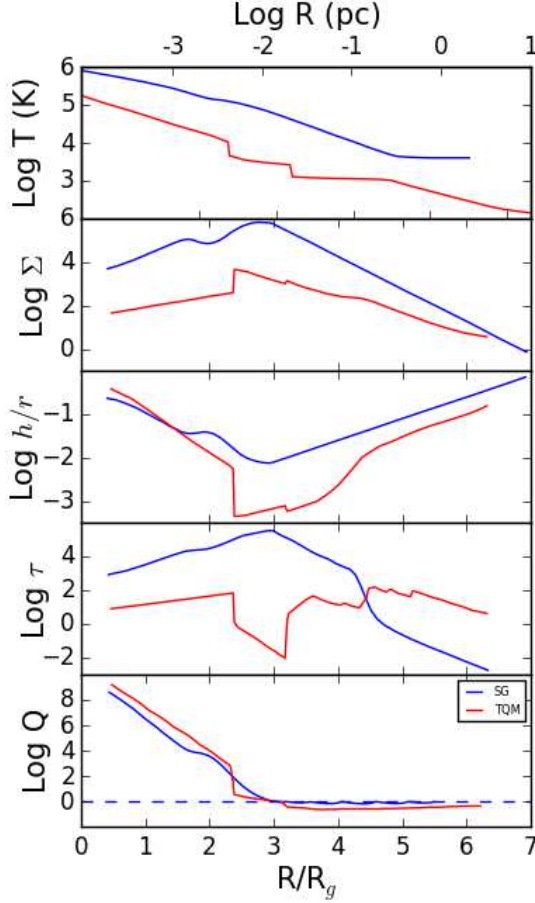


FIG. 1.— Models of accretion disks from SG (blue) and TQM (red). From top to bottom, we show temperature, surface density (in g cm^{-2}), disk aspect ratio h/r , optical depth τ , and Toomre Q vs radius. The top axis represents the translation from gravitational radius to parsecs for a $10^8 M_\odot$ SMBH.

and the assumed dynamics of the inflow are the root cause of these differences. SG assume that a constant turbulent viscosity drives the inflow; while TQM assume the inflow speed is a constant fraction of the local sound speed. In both cases the high Thompson scattering opacity from electrons produced by the ionization of hydrogen causes the inner disk to be optically thick. At intermediate radii, where the electron density drops precipitously, the opacity drops correspondingly, allowing the disk to cool and become thinner. At larger radii, where the temperature is low enough for dust grains to survive, dust opacity becomes important in the disk, so the disk again thickens and cools further. The disk masses (integrated out to 1 pc) are $3.7 \times 10^7 M_\odot$ and $6.5 \times 10^6 M_\odot$ for SG and TQM, respectively.

3. RESULTS

Figure 2 shows the result of calculating the torques from Equation (5) in a disk with the profile given by SG around a $10^8 M_\odot$ SMBH for a migrator of mass $100 M_\odot$. The figure shows the absolute value of the torque vs radius; black lines represent negative torque, and thick red lines represent positive torque. The spikes mark the points where the torque crosses zero. The direction of the torque is also given by arrows for clarity. These high-

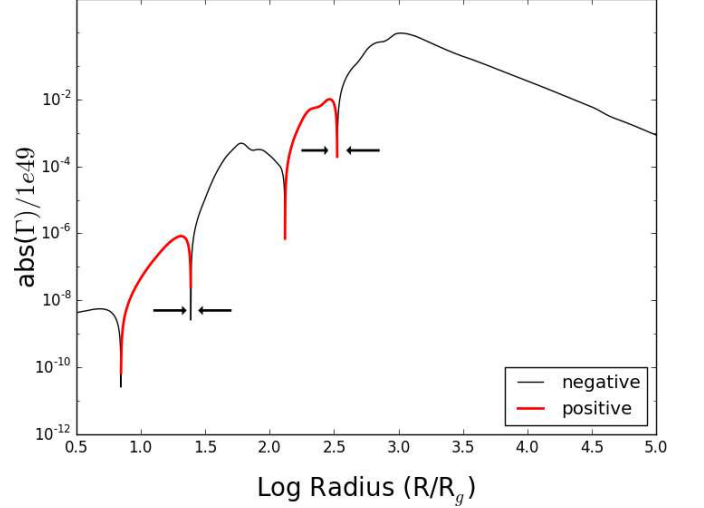


FIG. 2.— The absolute value of the torque Γ for the SG model, scaled by a factor of $10^{49} \text{ g cm}^2 \text{ s}^{-2}$, vs. gravitational radius R_g . Black lines indicate where the torque is negative, and thick red lines where it is positive. The arrows point in the direction of the torque, and show that inward- and outward-pointing torques meet at two of the zero-crossings, forming migration traps.

light the two migration traps in this disk model: one at $\log R = 1.39 R_g$, and the other at $\log R = 2.52 R_g$, corresponding to 24.5 and $331 R_g$, or 0.0004 and 0.003 pc for a $10^8 M_\odot$ SMBH. The Toomre Q parameters at the trap locations are $\sim 10^5$ and 16 , respectively, indicating that these regions are quite stable.

These estimates are for a fiducial value of $M_{\text{SMBH}} = 10^8 M_\odot$ and mass ratio $q = 10^{-6}$. However, we repeated our calculations for a range of each value ($5 \times 10^5 < M_{\text{SMBH}} < 5 \times 10^9 M_\odot$ and $0.1 < q < 10^{-6}$) and found no difference in the radial location of the migration traps in terms of R_g . We should expect this result, since the variables that depend on the mass ratio q and M_{SMBH} are Γ_0 and Θ , as seen in equations (4) and (6). These mass adjustments change the magnitude of the torques but not their radial position; i.e. the trap locations are not affected. However, the SG model assumes a particular value of M_{SMBH} . As we do not have access to their full set of models, we are unable to vary the black hole mass self-consistently in our calculations.

Figure 3 shows the results for the same torque calculation using the TQM model. We find one migration trap, at $\log R = 2.39 R_g$ ($245 R_g$, or 0.002 pc for a $10^8 M_\odot$ SMBH). At this radius, $Q = 3.5$. This trap occurs precisely at the point where the disk profiles are vertical, and the derivative is undefined (see Figure 1). To explore the robustness of this result, we made the profile differentiable by shifting the endpoints of each vertical section of the profile to vary the slope. In the extreme case, we adjusted the surface density profile to effectively round off the sharp peak at $\log R = 2.4 R_g$. Regardless of these changes, the migration trap continues to exist at the point where the surface density slope changes from positive to negative. Significantly, migration traps also exist in the SG model at the same locations—the points where the slope shifts from positive to negative, indicating that the slope change of the surface density profile is a key factor in determining where migration traps exist in these models (see also Masset et al. (2006)).

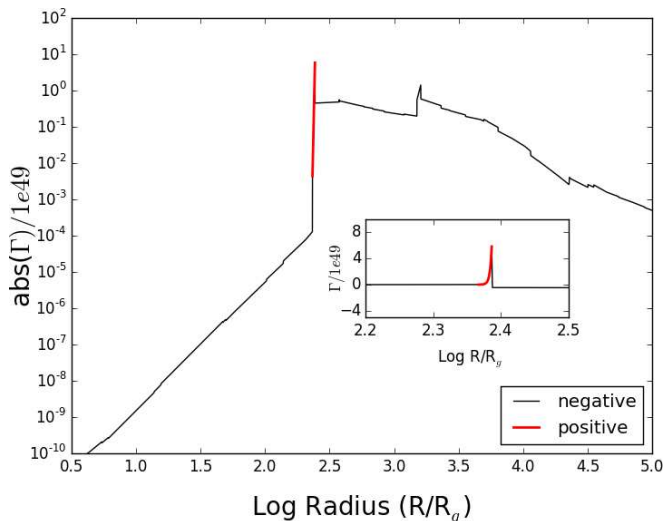


FIG. 3.— The absolute value of the torque Γ for the TQM model, scaled by a factor of $10^{49} \text{ g cm}^2 \text{ s}^{-2}$, vs. normalized radius R/R_g . Black lines indicate where the torque is negative, and red thick lines where it is positive. The inset shows Γ on a linear scale for a small region to better visualize the migration trap.

Note that in Figure 1 there is a small surface density discontinuity at $\log R \sim 3.2R_g$; however it does not yield a migration trap in Figure 3. Again we adjusted the endpoints of the vertical section of the profile to verify the robustness of this result. We found that the magnitude of the vertical change in the profile was insufficient to cause the torque to change sign. Thus, both a slope change *and* a large change in magnitude of the surface density of an AGN disk appear to be needed in order to create a migration trap.

4. IMPLICATIONS

The occurrence of migration traps in simple models of AGN disks implies that IMBH may form efficiently and quickly due to stellar black hole collisions at such locations, by analogy with giant planet core formation at migration traps in protoplanetary disks (Horn et al. 2012). Ignoring migration traps, McKernan et al. (2012) conservatively predict that a $10M_\odot$ black hole around a 10^8M_\odot SMBH can double its mass via collisions and gas accretion in 10 Myr. However, including migration traps can boost the collision rate of disk objects by more than a factor of 100. For a migrator at 10^4R_g in a migration trap with enhanced surface density of compact objects of $\Sigma_0 = 350 \text{ g cm}^{-2}$, assuming a reasonable distribution of eccentricities, the growth rate can reach over $dM/dt \sim 10^{-5}M_\odot \text{ yr}^{-1}$, which would result in a $10M_\odot$ black hole growing to $\sim 100M_\odot$ in 10 Myr. We also point out that the build-up of the IMBH (i.e. the merging of stellar mass black holes) is detectable in the local universe with LIGO. Nearby quasars such as Mrk 231 are good candidates to model and search for such events.

If this predicted growth occurs, there are observable implications, both in electromagnetic and gravitational radiation. For example, if the IMBH to SMBH mass ratio becomes large enough ($q \geq 10^{-4}$) a gap can form in the disk at the migration trap radius, leading to a flux decrement in the optical/UV disk SED. (Tanaka et al. 2011; Gültekin & Miller 2012; McKernan et al. 2014).

If the IMBH migrates into the central SMBH, a robust gravitational wave signal could be detected. Such a

scenario is more likely for a lower mass ($M < 10^{7.5}M_\odot$) primary or closer-in ($200R_g$) migrator. A binary system of mass $M_b = M_1 + M_2$ will decay via gravitational wave emission on a timescale (Peters 1964)

$$\tau_{\text{GW}} \approx \frac{5}{128} \frac{c^5}{G^3} \frac{a_b^4}{M_b^2 \mu_b} (1 - e_b^2)^{7/2}, \quad (8)$$

where the binary reduced mass $\mu_b = M_1 M_2 / M_b$, the binary semi-major axis is a_b , and its eccentricity $e_b \approx 0$. Rewriting in terms of M_1, M_2, a_b and normalized by $R_{g1} = 2GM_1/c^2$, the gravitational radius of the primary, yields

$$\tau_{\text{GW}} \approx 0.01 \text{ Myr} \left(\frac{M_1}{10^6 M_\odot} \right)^2 \left(\frac{M_2}{10^3 M_\odot} \right)^{-1} \left(\frac{a_b}{200 R_{g1}} \right)^4. \quad (9)$$

For a fiducial AGN disk lifetime of ~ 10 Myr, an IMBH formed at $200R_g$ in a disk around a SMBH with $M < 10^{7.5}M_\odot$ should merge with the primary within the disk lifetime. If such mergers are common, detectable gravitational wave events will be more frequent than previously supposed (e.g. Babak et al. 2008) and can be observed by the planned *LISA* mission (see also Holley-Bockelmann et al. 2010), with a complementary electromagnetic counterpart observable via oscillations in the FeK α line (McKernan et al. 2013; McKernan & Ford 2015).

On the other hand, IMBHs that form around more massive SMBHs, or more than twice as far away in the disk will outlast the AGN disk and survive, potentially until the next accretion episode. Such a binary system can affect the galactic bulge, scattering stars and altering the potential well. The IMBH can also itself grow due to gas accretion (Farris et al. 2014), changing the mass ratio of the system and possibly being visible as a mini-quasar with shifting radial velocities. We will return to some of these consequences in future work.

5. SUMMARY

Migration traps are equilibrium orbits in disks where regions of outward migration meet regions of inward migration. We study migration of massive objects in AGN accretion disks to determine whether migration traps exist in such environments. We examine two different steady-state, analytic models of AGN disks, and find that, despite the different assumptions used, migration traps occur in both models. These migration traps occur at locations of significant change in both the magnitude and gradient of the surface density. In the traps, massive objects, such as stellar mass black holes, can accumulate and merge, resulting in the formation of IMBHs. These IMBHs could ultimately clear out a gap in the accretion disk, producing multiple observable signatures (McKernan et al. 2014). The buildup of IMBHs could be a significant gravitational wave source for LIGO, and mergers of these IMBHs with their central SMBHs would increase the number and strain amplitude of expected gravitational wave sources detectable by *eLISA*.

Our prediction is based on analytical models that neglect evolution and make strong simplifying assumptions about the dynamics. Further studies may need to include effects shown to be important in the protoplan-

etary context, including torques due to magnetic fields (Guilet et al. 2013), and accretion heating feedback from the migrator (Benítez-Llambay et al. 2015). Accretion disk dynamics are more complex than the assumptions of either SG or TQM, as can be seen from the substantial differences between the models. Ultimately, migration depends on the detailed physical state of the disk, including the temperature, density, opacity, and turbulence. We therefore stress that our results should not be interpreted literally, but rather as a promising possibility worthy of further detailed modeling. We also assume that trapped compact objects will have common orbits with low eccentricity, and merge without any dynamical consequences. Further studies must determine whether collisions of migrators will perturb the disk and affect mi-

gration, and whether the scattering of compact objects will result in ejections from the disk and prevent our scenario entirely. A full, three-dimensional, time-evolving model will ultimately be needed in order to make robust predictions of whether AGN disks can efficiently form IMBHs within migration traps.

Thanks to Alex Hubbard, Yuri Levin, Cole Miller, Shane Davis, and the anonymous referee for useful discussions. JMB acknowledges generous support from the Helen Gurley Brown Trust. M-MML acknowledges support from NSF grant AST-1109395 and the Alexander von Humboldt Foundation. BM and KESF acknowledge support from NSF PAARE grant AST-1153335 and NSF grant PHY-1125915.

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